EXAMPLE 12 Physical forces have magnitude and direction and may thus be represented by vectors. If several forces act at once on an object, the resultant force is represented by the sum of the individual force vectors. Suppose that forces $\mathbf{i}+\mathbf{k}$ and $\mathbf{j}+\mathbf{k}$ are acting on a body. What third force $\mathbf{F}$ must we impose to counteract the two—that is, to make the total force equal to zero?

SOLUTION The force $\mathbf{F}$ should be chosen so that $(\mathbf{i}+\mathbf{k})+(\mathbf{j}+\mathbf{k})+\mathbf{F}=\mathbf{0}$; that is, $\mathbf{F}=-(\mathbf{i}+\mathbf{k})-(\mathbf{j}+\mathbf{k})=-\mathbf{i}-\mathbf{j}-2\mathbf{k}$. (Recall that $\mathbf{0}$ is the *zero vector*, the vector whose components are all zero.) ▲

EXERCISES

1. Calculate $(3\mathbf{i}+2\mathbf{j}+\mathbf{k})\cdot(\mathbf{i}+2\mathbf{j}-\mathbf{k})$.

2. Calculate $\mathbf{a}\cdot\mathbf{b}$ where $\mathbf{a}=2\mathbf{i}+10\mathbf{j}-12\mathbf{k}$ and $\mathbf{b}=-3\mathbf{i}+4\mathbf{k}$.

3. Find the angle between $7\mathbf{j}+19\mathbf{k}$ and $-2\mathbf{i}-\mathbf{j}$ (to the nearest degree).

4. Compute $\mathbf{u}\cdot\mathbf{v}$, where $\mathbf{u}=\sqrt{3}\mathbf{i}-315\mathbf{j}+22\mathbf{k}$ and $\mathbf{v}=\mathbf{u}/\|\mathbf{u}\|$.

5. Is $\|8\mathbf{i}-12\mathbf{k}\|\cdot\|6\mathbf{j}+\mathbf{k}\|-|(8\mathbf{i}-12\mathbf{k})\cdot(6\mathbf{j}+\mathbf{k})|$ equal to zero? Explain.

In Exercises 6 to 11, compute $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, *and* $\mathbf{u}\cdot\mathbf{v}$ *for the given vectors in* $\mathbb{R}^3$.

6. $\mathbf{u}=15\mathbf{i}-2\mathbf{j}+4\mathbf{k}$, $\mathbf{v}=\pi\mathbf{i}+3\mathbf{j}-\mathbf{k}$

7. $\mathbf{u}=2\mathbf{j}-\mathbf{i}$, $\mathbf{v}=-\mathbf{j}+\mathbf{i}$

8. $\mathbf{u}=5\mathbf{i}-\mathbf{j}+2\mathbf{k}$, $\mathbf{v}=\mathbf{i}+\mathbf{j}-\mathbf{k}$

9. $\mathbf{u}=-\mathbf{i}+3\mathbf{j}+\mathbf{k}$, $\mathbf{v}=-2\mathbf{i}-3\mathbf{j}-7\mathbf{k}$

10. $\mathbf{u}=-\mathbf{i}+3\mathbf{k}$, $\mathbf{v}=4\mathbf{j}$

11. $\mathbf{u}=-\mathbf{i}+2\mathbf{j}-3\mathbf{k}$, $\mathbf{v}=-\mathbf{i}-3\mathbf{j}+4\mathbf{k}$

12. Normalize the vectors in Exercises 6 to 8. (Only the solution corresponding to Exercise 7 is in the Student Guide.)

13. Find the angle between the vectors in Exercises 9 to 11. If necessary, express your answer in terms of $\cos^{-1}$.

14. Find the projection of $\mathbf{u}=-\mathbf{i}+\mathbf{j}+\mathbf{k}$ onto $\mathbf{v}=2\mathbf{i}+\mathbf{j}-3\mathbf{k}$.

15. Find the projection of $\mathbf{v}=2\mathbf{i}+\mathbf{j}-3\mathbf{k}$ onto $\mathbf{u}=-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

16. What restrictions must be made on the scalar b so that the vector $2\mathbf{i}+b\mathbf{j}$ is orthogonal to (a) $-3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and (b) $\mathbf{k}$?

17. Find two nonparallel vectors both orthogonal to $(1, 1, 1)$.